- A Markov chain is a mathematical model for a process which moves step by step through various states.
- In a Markov chain, the probability that the process moves from any given state to any other particular state is always the same , regardless of the history of the process
- A Markov chain consists of states and transition probabilities.
- Each transition probability is the probability of moving from one state to another in one step.
- The transition probabilities are independent of the past and depend only on the two states involved.
- The matrix of transition probabilities is called the transition matrix.
- Markov Modeling is an extremely important tool in the field of modeling and analysis of telecommunication networks.
- Example: Markov Models are applicable in the following networking problems
    - Connection admission control(CAC)
    - Bandwidth Allocation
    - Routing
    - Queuing and scheduling

Markov Chain

- A Markov chain is a sequence of random values whose probabilities at a time interval depends upon the value of the number at the **previous** time.
- A simple example is the non returning random walk, where the walkers are restricted to not go back to the location just previously visited.
- The controlling factor in a Markov chain is the **transition probability**, it is a conditional probabilty for the system to go to a particular new state, given the current state of the system.
- For many problems, such as simulated annealing, the Markov chain obtains the much desired importance sampling.
- This means that we get fairly efficient estimates if we can determine the proper transition probabilities.
- Markov chains can be used to solve a very useful class of problems in a rather remarkable way.

Some Background Information

- Mathematical models that evolve over time in a probabilistic manner are called stochastic processes.

- A special kind of stochastic process is a Markov Chain, where the outcome of an experiment depends only on the outcome of the previous experiment.

Why Study Markov Chains?

Markov chains are used to analyze trends and predict the future. (Weather, stock market, genetics, product success, etc.

Key Features of Markov Chains

A sequence of trials of an experiment is a **Markov chain** if

- the outcome of each experiment is one of a set of discrete states.
- the outcome of an experiment depends only on the present state, and not on any past states.
- the transition probabilities remain constant from one transition to the next.

Internet application

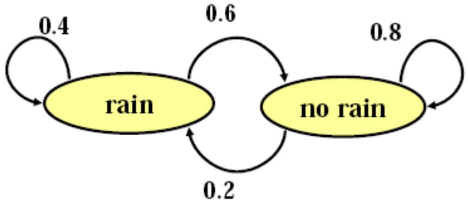
- Markov models have also been used to analyze web navigation behavior of users.
- A user's web link transition on a particular website can be modeled using first- or second order.
- Markov models and can be used to make predictions regarding future navigation and to personalize the web page for an individual user.

Examples of Markov Chain

1.) Weather

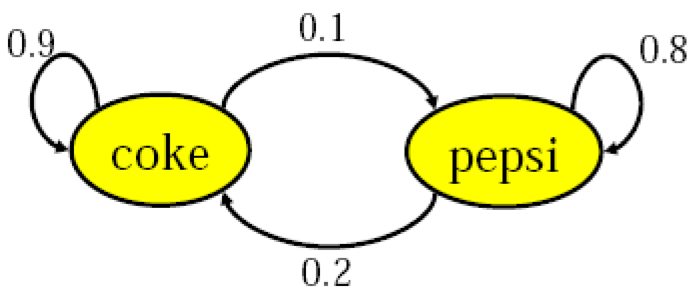
   a. If it rains today there is 40% probability of raining tomorrow
   b. If it doesnot rains today there is 20% probability of raining tomorrow

Transition Matrix = $\begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$



2.) Coke and Pepsi Example

a. If a person purchase coke now the probability of purchase of coke next time is 90%
b. If a person purchase pepsi now the probability of purchasing pepsi next time is 80%



Transition Matrix = $\begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$

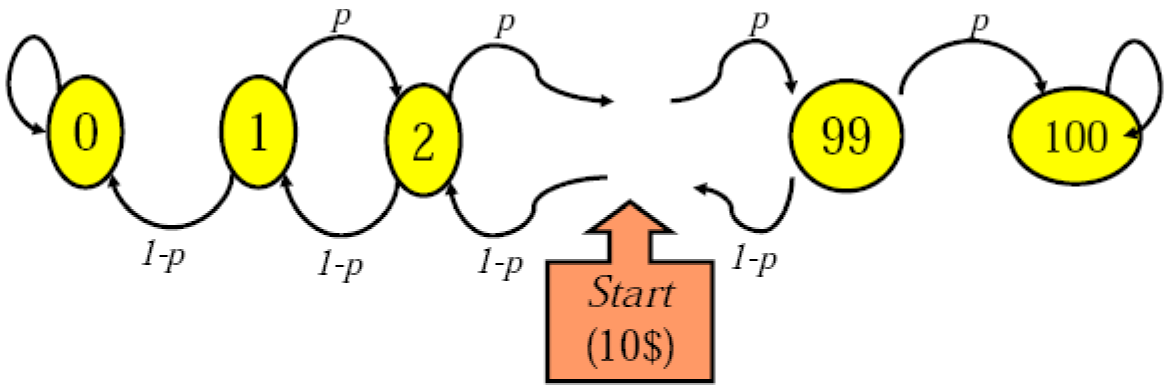
Probability of purchasing coke in 3rd purchase ???

Pr[Pepsi->?->Coke] = Pr[Pepsi->coke->Coke] + Pr[Pepsi->pepsi->Coke]

0.2*0.9 + 0.8*0.2

Probability= $\begin{pmatrix} 0.9 & 0.1 \\ \boldsymbol{0.2} & \boldsymbol{0.8} \end{pmatrix} \begin{pmatrix} \boldsymbol{0.9} & 0.1 \\ \boldsymbol{0.2} & 0.8 \end{pmatrix} = \begin{pmatrix} 0.83 & 0.17 \\ & \underline{\boldsymbol{0.34}} \end{pmatrix}$ 0.66

3.) Gambler's Example


p
p
p
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1
2
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100
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Start
(10$)