Testing for auto-correlation

- The uniformity test of random numbers is only a necessary test for randomness, not sufficient one.
- A sequence of numbers may be perfectly uniform and still not random.

Example

- Let us consider the following random numbers

49, 95, 82, 19, 41, 31, 12, 53, 62, 40, 87, 83,
26, 01, 91, 55, 38, 75, 90, 35, 71, 57, 27, 85,
52, 08, 35, 57, 88, 38, 77, 86, 29, 18, 09, 96,
58, 22, 08, 93, 85, 45, 79, 68, 20, 11, 78, 93,
21, 13, 06, 32, 63, 79, 54, 67, 35, 18, 81, 40,
62, 13, 76, 74, 76, 45, 29, 36, 80, 78, 95, 25,
52

Example

- These 73 random numbers giving 72 pairs, are grouped into 9 classes with expectation of 8 in each group.

Class	Frequency	Diff	$(Diff)^2$
R1≤.33 & r2≤.33	9	1	1
R1≤.67 & r2≤.33	7	1	1
R1≤ 1.0 & r2≤.33	6	2	4
R1≤.33 & r2≤.67	6	2	4
R1≤.67 & r2≤.67	8	0	0
R1≤1.0& r2≤0.67	9	1	1
R1≤.33 & r2≤1.0	7	1	1
R1≤.67 & r2≤1.0	9	1	1
R1≤1.0 & r2≤1.0	11	3	9
	72		22

Example

- Chi-square=22/8=2.75 ;Degrees of freedom are (n-2);9-2=7(Seven)
- The criterion value of chi-square for seven degrees of freedom at 95% confidence level is 14.067.
- The value of Chi-square obtained for the given set of random numbers is well within the acceptable limit, and hence , they are not serially auto correlated.

Problem

- A sequence of random numbers is given below. Use chi-square test with $\alpha=0.05$ to test whether these numbers are uniformly distributed and serial autocorrelation.

07, 05, 96, 14, 10, 90, 21, 15, 84, 28, 20, 78, 35, 25, 72, 42, 30, 66, 49, 35, 60, 56, 40, 54, 63, 45, 48, 70, 50, 42, 77, 55, 36, 84, 60, 30, 91, 65, 24, 98, 70, 18, 07, 75, 12, 14, 80, 06, 21, 85, 96, 28, 90, 35, 95, 84, 42, 05, 78, 49, 10, 72, 56, 15, 66, 63, 20, 60, 70, 25, 54, 77, 30, 48, 84, 35, 42, 91, 40, 36, 98, 45, 30, 07, 50, 24, 14, 55, 18, 21

Poker Test

- This test gets its name from a game of cards called poker
- This test not only tests the randomness of the sequence of numbers, but also the digits comprising of each number
- Every random number of five digits or every sequence of five digits is treated as poker hard.

Poker Test

- 71549 are five different digits
- 55137 would be pair
- 33669 would be two pairs
- 55513 would be three of a kind
- 44477 would be a full house
- 77774 would be four of a kind
- 88888 would be five of a kind

The occurrence of five of a kind is rare.

Poker Test

- In 10,000 random and independent numbers of five digits each, you may be expect the following distribution of various combinations.

Five different digits	3024 or 30.24%
pairs	5040 or 50.40 %
Two-pairs	1080 or 10.80 %
Three of a kinds	720 or 7.20 %
Full houses	90 or 0.90 %
Four of a kinds	45 or 0.45 %
Five of a kinds	1 or 0.01 %

Poker Test

- *Poker Test* - based on the frequency with which certain digits are repeated.

  Example:

  0.255 0.577 0.331 0.414 0.828 0.909

  Note: a pair of like digits appear in each number generated.

Poker Test

In 3-digit numbers, there are only 3 possibilities.

P(3 different digits) = (2nd diff. from 1st) * P(3rd diff. from 1st & 2nd)

= (0.9) (0.8) = 0.72

P(3 like digits) = (2nd digit same as 1st) * P(3rd digit same as 1st)

= (0.1) (0.1) = 0.01

P(exactly one pair) = 1 - 0.72 - 0.01 = 0.27

Poker test

(Example)

A sequence of 1000 three-digit numbers has been generated and an analysis indicates that 680 have three different digits, 289 contain exactly one pair of like digits, and 31 contain three like digits. Based on the poker test, are these numbers independent?

Let $\alpha = 0.05$.

The test is summarized in next table.

Poker Test

Combination, i	Observed Frequency, O_i	Expected Frequency, E_i	$\frac{(O_i - E_i)^2}{E_i}$
Three different digits	680	720	2.24
Three like digits	31	10	44.10
Exactly one pair	289	270	1.33
	1000	1000	47.65

The appropriate degrees of freedom are one less than the number of class intervals. Since $\chi^2_{0.05, 2} = 5.99 < 47.65$, the independence of the numbers is rejected on the basis of this test.