Continuous system simulation

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Continuous system

- If the dependent variable or any of its derivatives appear in any other form, such as being raised to a power, or are combined in any other way for example, by being multiplied together, the differential equation is said to be nonlinear.
- When more than one independent variable occurs in a differential equation, the equation is said to be partial differential equation.
- It can involve the derivatives of the same dependent variable with respect to each of the independent variables.

Analog Computer

- The most widely used form of analog computer is the electronic analog computer, based on the use of high gain dc (direct current) amplifiers, called operational amplifiers.
- Voltages in the computer are equated to mathematical variables, and the operational amplifiers can add and integrate the voltages.
- With appropriate circuits, an amplifier can be made to add several input voltages, each representing a variable of the model, to produce a voltage representing the sum of the input variables.

Analog Computer

- Different scale factors can be used on the inputs to represent coefficients of the model equations.
- Such amplifiers are called summers.
- Another circuit arrangement produces an integrator for which the output is the integral with respect to time of a single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- To satisfy the equations of the model, it is sometimes necessary to use a sign inverter, which is an amplifier designed to cause the output to reverse the sign of the input.

Analog Computer

- Electronic analog computers are limited in accuracy for several reasons.
- It is difficult to carry the accuracy of measuring a voltage beyond a certain point.
- Secondly, a number of assumptions are made in deriving the relationships for operational amplifiers, none of which is strictly true; so, amplifiers do not solve the mathematical model with complete accuracy.
- A particularly troublesome assumption is that there should be zero output for zero input.
- Another type of difficulty is presented by the fact that the operational amplifiers have a limited dynamic range of output, so that scale factors must be introduced to keep within the range.

Analog Computer

- As a consequence, it is difficult to maintain an accuracy better than 0.1% in an electronic analog computer.
- Other forms of analog computers have similar problems and their accuracies are not significantly better.
- A digital computer is not subject to the same type of inaccuracies.
- Virtually any degree of accuracy can be programmed and, with the use of floating-point representation of numbers, an extremely wide range of variations can be tolerated.
- Integration of variables is not a natural capability of a digital computer, as it is in an analog computer, so that integration must be carried out by numerical approximations.

Analog Computer

- However, methods have been developed which can maintain a very high degree of accuracy.
- A digital computer also has the advantage of being easily used for many different problems.
- An analog computer must usually be dedicated to one application at a time, although time-sharing sections of an analog computer has become possible.
- In spite of the widespread availability of digital computers, many users prefer to use analog computers. There are several considerations involved.

Analog Computer

- The analog representation of a system is often more natural in the sense that it directly reflects the structure of the system; thus simplifying both the setting—up of a simulation and the interpretation of the results.

- Under certain circumstances, an analog computer is faster than a digital computer, principally because it can be solving many equations in a truly simultaneous manner; whereas a digital computer can be working only on one equation at a time, giving the appearance of simultaneity by interfacing the equations.

- On the other hand, the possible disadvantages of analog computers, such as limited accuracy and the need to dedicate the computer to one problem, may not be significant.

Analog methods

- The general method by which analog computers are applied can be demonstrated using the second—order differential equation that has already been discussed:

- Solving the equation for the highest order derivative gives

$$M\frac{d^2x}{dy}+D\frac{dx}{dy}+Kx=KF(t)$$

$$M\frac{d^2x}{dy}=KF(t)-D\frac{dx}{dy}-Kx$$

Analog methods

- Suppose a variable representing the input F(t) is supplied, and assume for the time being that there exist variables representing —x and $\frac{dx}{dy}$
- These three variables can be scaled and added with a summer to produce a voltage representing MX.
- Integrating this variable with a scale factor of 1/M produces $\frac{dx}{dy}$
- Changing the sign produces —x, which supplies one of the variables initially assumed; and a further integration produces —x, which was the other assumed variable.

Analog methods

- For convenience, a further sign inverter is included to produce $+x$ as an output.
- A block diagram to solve the problem in this manner is shown in following Figure.

Analog methods

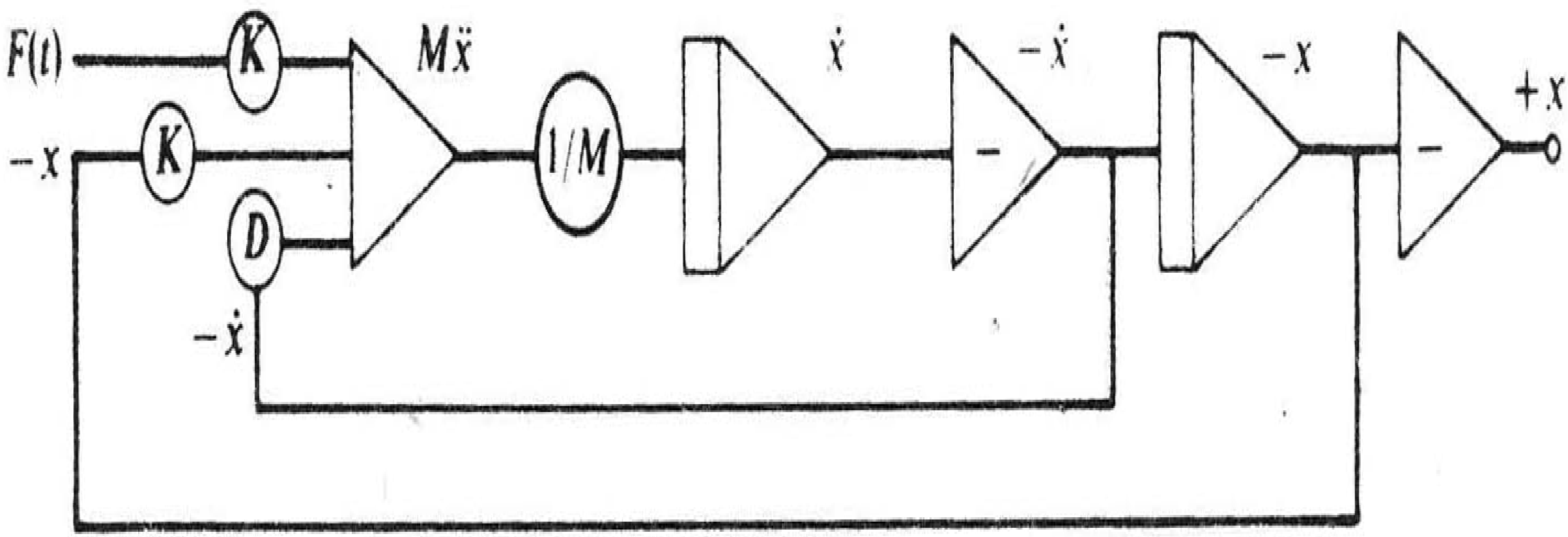


Diagram for the automobile suspension problem

Analog methods

- The symbols used in the figure are standard symbols for drawing block diagrams representing analog computer arrangements.
- The circles indicate scale factors applied to the variables.
- The triangular symbol at the left of the figure represents the operation of adding variables.
- The triangular symbol with a vertical bar represents an integration, and the one containing a minus sign is a sign changer.

Analog methods

- The addition on the left, with its associated scaling factors, corresponds to the addition of the variables representing the three forces on the wheel, producing a variable representing M .
- The scale is changed to produce $\frac{d^2x}{dy}$ and the result is integrated twice to produce both $\frac{dx}{dy}$ and x.
- Sign changers are introduced so that variables of the correct sign can be fed back to the adder, and the output can be given in convenient form.
- With an electronic analog computer, the variables that have been described would be voltages, and the symbols would represent operational amplifiers arranged as adders, integrators, and sign changers.

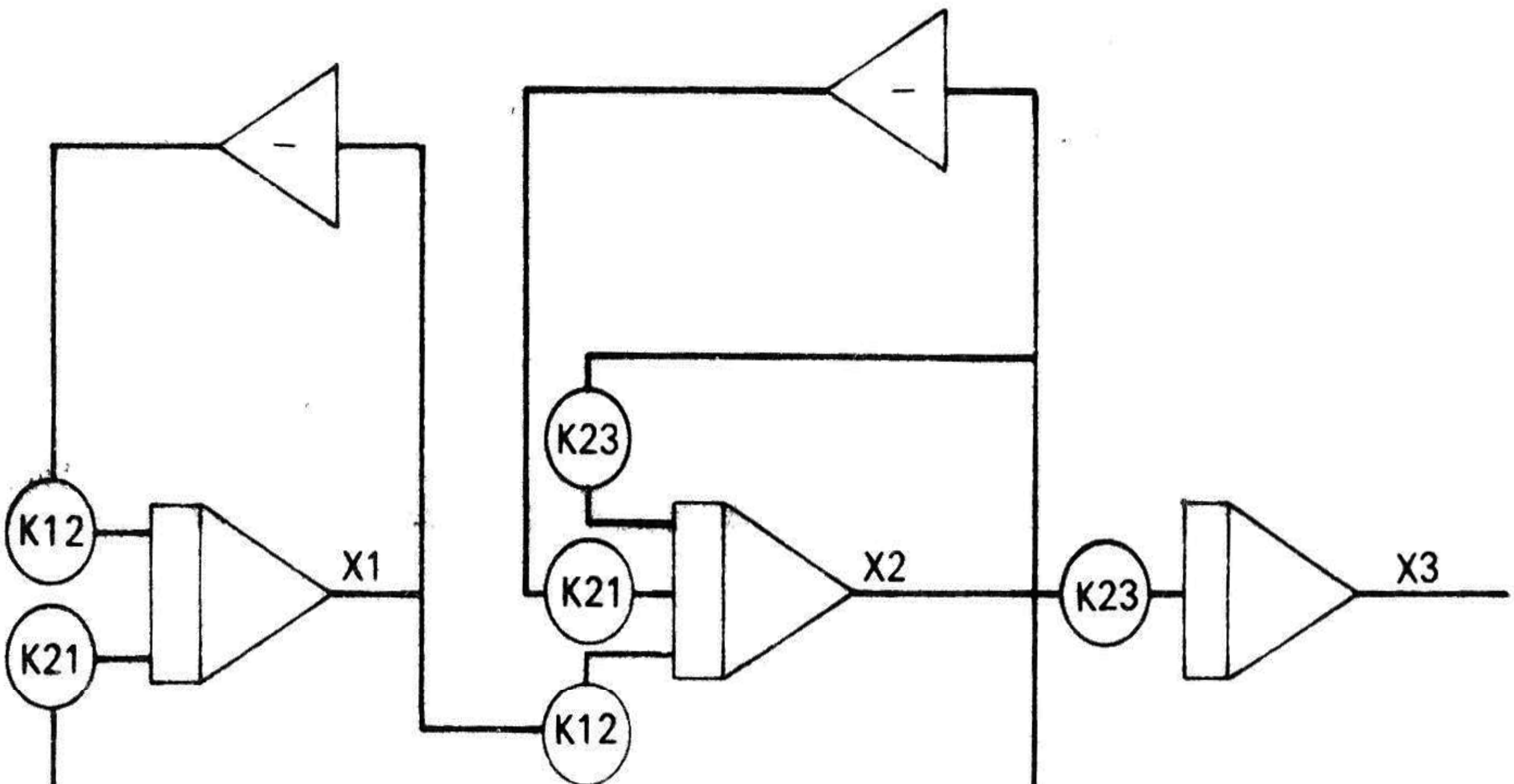
Analog methods

- The above Figure would then represent how the amplifiers are interconnected to solve the equation.
- It should be pointed out, however, that there can be several ways of drawing a diagram for a particular problem, depending upon which variables are of interest, and on the size of the scale factors.
- When a model has more than one independent variable, a separate block diagram is drawn for each independent variable and, where necessary, interconnections are made between the diagrams.

Analog methods

- As an example, the following Figure shows a block diagram for solving the model of the liver.

Analog methods



Analog computer model of the liver

Analog methods

- There are three integrators, shown at the bottom of the figure. Reading from left to right, they solve the equations for X_1, X_2, X_3.
- Interconnections between the three integrators, with sign changers where necessary, provide inputs that define the differential coefficients of the three variables.
- The first integrator, for example, is solving the equation

$$\frac{dx_1}{dy} = -k_{12}x_1 + k_{21}x_2$$

Analog methods

- The second integrator is solving the equation

$$\frac{dx_2}{dy} = k_{12}x_1 - (k_{21} - k_{23})x_2$$

- The last integrator solves the equation

$$\frac{dx_3}{dy} = k_{23}x_2$$